

4.4 第一类换元积分法

一、填空题

1. 设 $f(x)$ 是连续函数, 则 $d\int f(x)dx = \underline{\hspace{2cm}}$, $\int df(x) = \underline{\hspace{2cm}}$,

$\frac{d}{dx}\int f(x)dx = \underline{\hspace{2cm}}$, $\int f'(x)dx = \underline{\hspace{2cm}}$ (其中 $f'(x)$ 连续);

2. $a^{3x} dx = \underline{\hspace{2cm}} d(a^{3x} - 1)$;

3. $\frac{x}{x^2 - 1} dx = \underline{\hspace{2cm}} d\ln(x^2 - 1)$;

4. $\frac{dx}{1 + 9x^2} = \underline{\hspace{2cm}} d(\arctan 3x)$;

5. $\sin 2x dx = \underline{\hspace{2cm}} d(1 - 4\cos 2x)$;

6. $(3 - x)dx = \underline{\hspace{2cm}} d[(3 - x)^2 - 4]$.

7. $dx = \underline{\hspace{2cm}} d(2 - 3x)$;

8. $xdx = \underline{\hspace{2cm}} d(2x^2 - 1)$;

9. $\frac{1}{x} dx = d \underline{\hspace{2cm}}$

10. $\frac{\ln x}{x} dx = \ln x d \underline{\hspace{2cm}} = d \underline{\hspace{2cm}}$

11. $\sin \frac{x}{3} dx = \underline{\hspace{2cm}} d(\cos \frac{x}{3})$;

12. $xe^{-2x^2} dx = d \underline{\hspace{2cm}}$;

13. $\frac{1}{1 + 9x^2} dx = \underline{\hspace{2cm}} d(\arctan 3x)$;

14. $\frac{xdx}{\sqrt{1 - x^2}} = \underline{\hspace{2cm}} d(\sqrt{1 - x^2})$;

二、求下列不定积分

1. $\int \frac{x}{\sqrt{2 - 3x^2}} dx$;

2. $\int xe^{-x^2} dx$;

3. $\int \frac{3x^3}{1 - x^4} dx$;

4. $\int e^{-5x} dx$;

5. $\int \frac{e^x}{1 + e^x} dx$;

6. $\int (2x + 1)^{10} dx$;

7. $\int \frac{\sin x}{\cos^3 x} dx$;

8. $\int \sin 2x dx$;

9. $\int e^{3x} dx$;

10. $\int \sqrt{1 - 2x} dx$;

11. $\int (x^2 - 3x + 1)^{100} (2x - 3) dx$;

12. $\int \frac{x^2}{(x - 1)^{100}} dx$;

$$13. \int \frac{1}{1+3x} dx ;$$

$$15. \int \frac{x \tan \sqrt{1+x^2}}{\sqrt{1+x^2}} dx ;$$

$$17. \int \sin^3 x \cos^5 x dx ;$$

$$19. \int \frac{\sin x}{1+\cos x} dx ;$$

$$14. \int \frac{1}{x \ln x \ln \ln x} dx ;$$

$$16. \int \frac{\sin x + \cos x}{(\sin x - \cos x)^3} dx ;$$

$$18. \int e^x \sin e^x dx ;$$

$$20. \int \sin 2x \cos 3x dx ;$$